Chapter 7: Induction Motors

Induction machines – rotor voltage **induced** in rotor windings. No need for physical wires or dc field current (like in a synchronous machine).

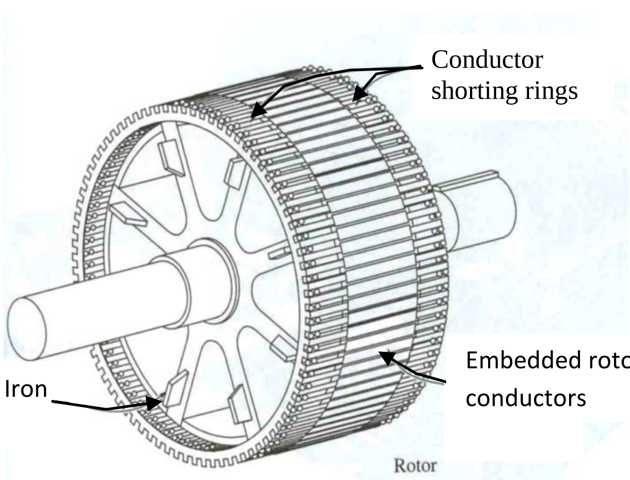
It can be motor or generator. But **rarely used as generator** due to many disadvantages (eg: it always consumes reactive power, low PF, not stand alone).

7.1. Induction motor (IM) construction

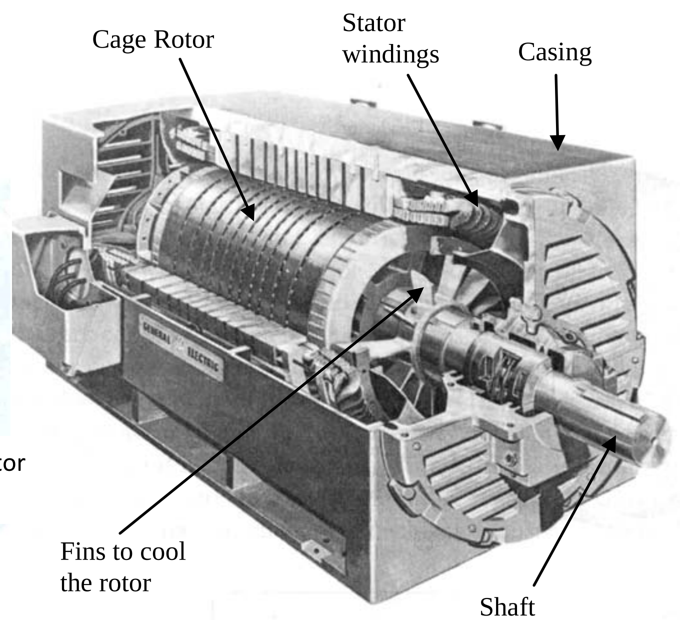
Induction motors have both a **stator and rotor**.

There are **two types of rotor** construction:

- **Cage rotor** – **conducting bars** are **laid in slots** carved in the face of the rotor and shorted at the end by **shorting (or end) rings**.

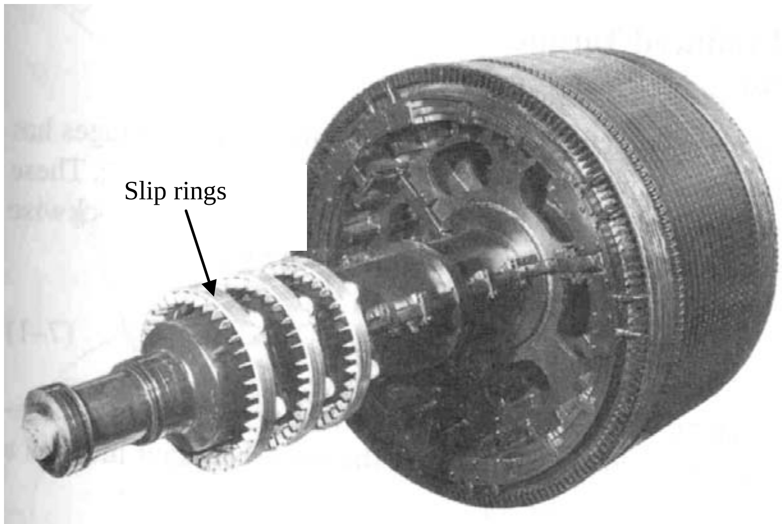


Sketch of cage rotor

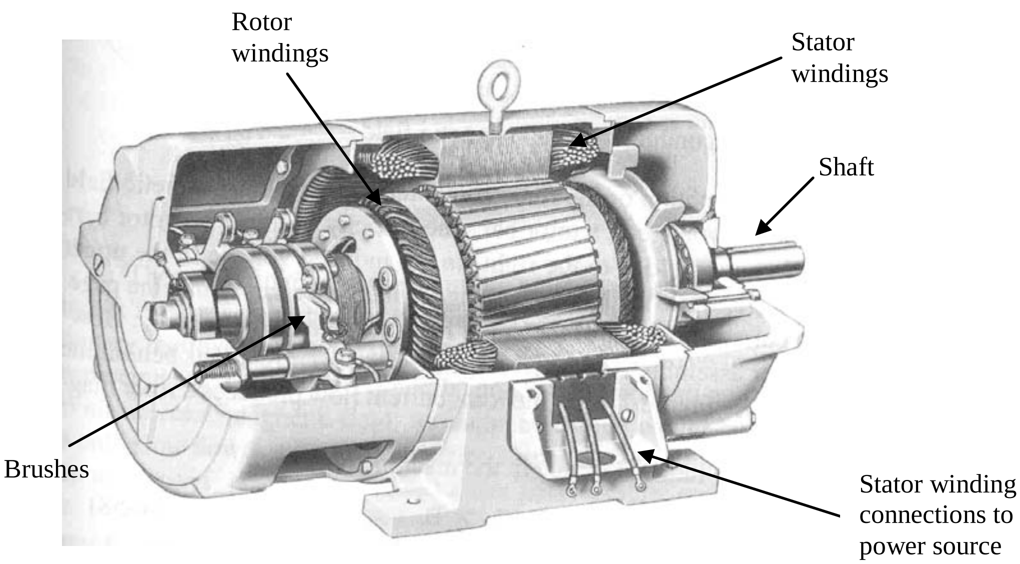


Typical large cage rotor

- **Wound rotor** – has a set of **3-phase windings**, usually **Y-connected** with the ends tied to **slip rings**. Rotor windings are **shorted by brushes** riding on the slip rings**.**



Typical wound rotor for induction motors.



Cutaway diagram of a wound rotor induction motor.

Wound rotor type is **more expensive –** require **more maintenance** due to **wear** associated with **brushes and slip rings**.

7.2. Basic induction motor concepts

The development of induced torque in an induction motor

When current flows in the 3-phase stator windings, a magnetic field $\bar{B}_S$ is produced. The **speed of magnetic field rotation** is given by:

$$n_{\text{sync}} = \frac{120f_e}{P}$$

where f_e = system frequency in Hz
P = number of poles in machine

The rotating magnetic field $\bar{B}_S$ will pass over the rotor bars causing **induced voltage** in them given by:

$$e_{\text{ind}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$$

Where $\bar{v}$ = velocity of the bar relative to the magnetic field
$\bar{B}$ = magnetic flux density
$\bar{l}$ = length of conductor in the magnetic field

Hence, **rotor currents** will flow which will **lag** behind e_{ind} due to the rotor having an inductive element.

This rotor current will then create a **rotor magnetic field $\bar{B}_R$**.

The **interaction between both magnetic fields** will produce **torque** in a **counterclockwise direction**:

$$\tau_{\text{ind}} = k\bar{B}_R \times \bar{B}_S$$

The induced torque will generate acceleration causing the **rotor to spin**.

However, there is **finite upper limit** to motor **speed**.

Reason for the finite upper limit

If **rotor speed = synchronous speed**

⇓

Rotor **bars appear stationary** relative to **the magnetic field**

⇓

Hence, $e_{\text{ind}} = 0$

⇓

No rotor current is present

⇓

Therefore, $\bar{B}_{\text{R}} = 0$

⇓

Since $\tau_{\text{ind}} = k\bar{B}_{\text{R}} \times \bar{B}_{\text{S}}$,
$\tau_{\text{ind}} = 0$

⇓

Rotor slows down due to friction

Conclusion**: Induction motor can speed up to *near synchronous speed* but never actually reach it.**

Note: Both $\bar{B}_{\text{S}}$ and $\bar{B}_{\text{R}}$ rotates at synchronous speed n_{sync} while rotor rotates at slower speed.

The concept of rotor slip

Rotor bar **induced voltage** is **dependent upon the speed** of **rotor relative to the magnetic fields**.

This can be easily termed as **slip speed**:

$$n_{\text{slip}} = n_{\text{sync}} - n_{\text{m}}$$

where n_{slip} = slip speed of the machine

n_{sync} = speed of magnetic field

n_{m} = mechanical shaft speed of motor

From this we can define **slip** (relative speed expressed on a percentage basis):

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}} \times 100$$

Slip can be expressed in terms of angular velocity, ω:

$$s = \frac{\omega_{\text{sync}} - \omega_{\text{m}}}{\omega_{\text{sync}}} \times 100\%$$

Notice:
- rotor rotates at synchronous speed, ***s* =**
- **rotor is stationary, *s* =**

The rotor mechanical speed can be obtained using:

$$n_{\text{m}} = (1 - s)n_{\text{sync}}$$

or

$$\omega_{\text{m}} = (1 - s)\omega_{\text{sync}}$$

Note: All normal motor speeds fall between s = 0 and s = 1.

The electrical frequency on the rotor

An induction motor is like a **rotating transformer**, i.e.

Stator (primary) induces voltage in the rotor (secondary)

However, in induction motor:

secondary frequency not necessarily the same as primary frequency

When **rotor is locked**, $\boldsymbol{n_m = 0}$ **r/min**,

$$s = 1$$
$$f_r = f_e$$

At **rotor** rotates **synchronous to field**, $\boldsymbol{n_m = n_{sync}}$,

$$s = 0$$

$$f_r = 0$$

Hence, at **other rotor speeds**, i.e. $\boldsymbol{0 < n_m < n_{sync}}$,

$$f_e < f_r < 0$$

By substituting for *s*,

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

Alternatively, since $n_{sync} = 120 f_e / P$,

$$f_r = \frac{P}{120} \quad (n_{sync} - n_m)$$

This shows that the **relative difference between synchronous speed and rotor speed** will **determine** the **rotor frequency**.

Example

A 208V, 7.46-kW, 4-pole, 60-Hz, Y-connected induction motor has a full load slip of 5%.

(a) What is the synchronous speed of the motor?

(b) What is the rotor speed of the motor at rated load?

(c) What is the rotor frequency of the motor at rated load?

(d) What is the shaft torque of this motor at rated load?

7.3. The equivalent circuit of an induction motor

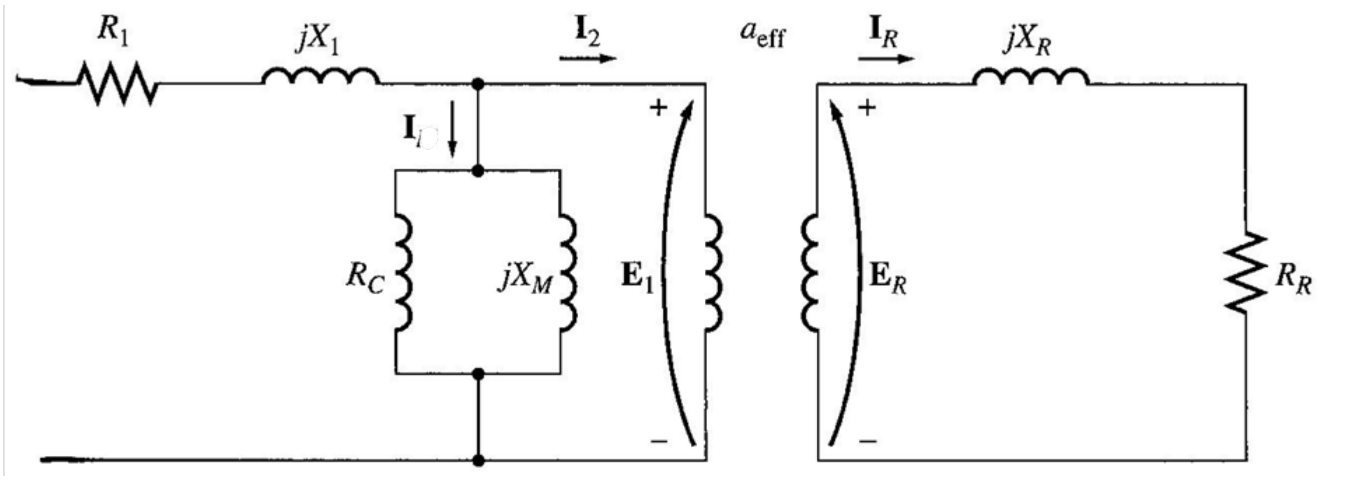
The **operation of induction motor** relies on

induction of **rotor voltages and currents** due to stator circuit, i.e. transformer action.

Hence, **induction motor equivalent circuit similar** to that of a transformer. So to achieve the final equivalent circuit, let's start with transformer model.

The transformer model of an induction motor

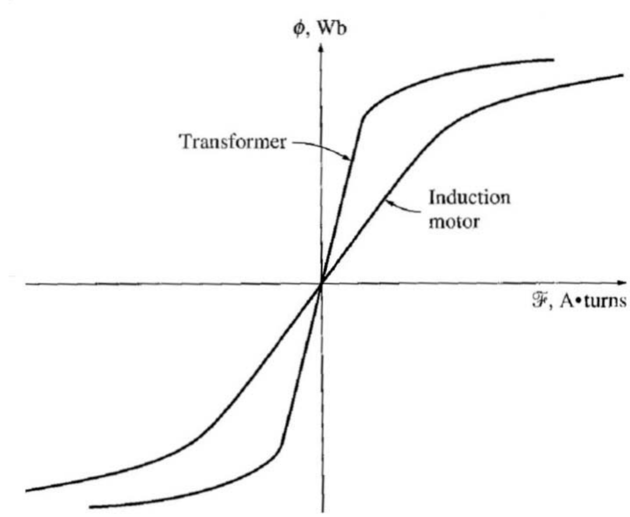
The transformer per-phase equivalent circuit **representing the operation of an induction motor** is:



The stator circuit model

- R_1 = Stator resistance
- E_1= Internal stator voltage
- X_1 = Stator leakage reactance
- The **flux in the IM** is **related** to the **integral** of the **applied voltage**, $\bar{E}_1$.

The IM *mmf* - ϕ curve (magnetisation curve) compared to similar curve for a transformer is shown on the right.



Slope of IM curve is **shallower** than the **transformer**.

⇨ due to the ____________________ in the induction motor.

The air gap results in a **higher reluctance path** which **requires** a **higher magnetising current** for a given flux level.

Therefore, X_{M} in **induction motor** is **smaller than** in an ordinary **transformer**.

- The **primary internal stator voltage** $\bar{E}_1$ is **coupled to the secondary** $\bar{E}_{\mathrm{R}}$ by an **ideal transformer** with an **effective turns ratio** $\boldsymbol{a_{eff}}$.

  For **wound rotor motor**,

$$a_{\mathrm{eff}} = k\left(\frac{\text{conductors per phase on stator}}{\text{conductors per phase on rotor}}\right)$$

  This equation is **not valid for cage rotor type** (because of no distinct rotor windings) **but $\boldsymbol{a_{\mathrm{eff}}}$ exists** in an IM.

- $\bar{E}_{\mathrm{R}}$ produces current flow in shorted rotor (or secondary) circuit.

The **primary impedances and magnetising current** in **IM** is very **similar** to corresponding **components** in a **transformer equivalent circuit**.

Difference:

Effect of __________________ **rotor frequency on rotor voltage $\bar{E}_{\mathrm{R}}$ and rotor impedances** R_{R} and jX_{R}.

The rotor circuit model

When **voltage** is **applied to the stator** windings,

⇨ **voltage** will be **induced** in the rotor circuit.

The amount of induced rotor voltage is **dependent upon slip**.

In general, **as the relative motion between the rotor and the stator magnetic fields increases, the resulting rotor voltage and rotor frequency increases.**

Hence, **when:**

Rotor stationary or locked or blocked (s = 1) $\Rightarrow$ Largest relative motion between rotor and $\bar{B}_{\mathrm{s}}$.

$\therefore$ **largest** $\bar{E}_{\mathrm{R}}$ and f_{r} induced.

Rotor at **near synchronous speed.** (s = 0) $\Rightarrow$ **Smallest** $\bar{E}_{\mathrm{R}}$ (0 V) and f_{r} (0 Hz) are induced.

Therefore, the magnitude of the induced voltage at any slip is:

$$E_{\mathrm{R}} = sE_{\mathrm{R0}}$$

where E_{R0}= induced rotor voltage at **locked-rotor** conditions

The frequency of the induced voltage at any slip is:

$$f_{\mathrm{r}} = sf_{\mathrm{e}}$$

The **rotor contains both resistance and reactanc**e. However, **only the reactance** will be **affected by** the **frequency of the rotor voltage and current**. Hence,

$$\begin{aligned} X_{\mathrm{R}} &= \omega_{\mathrm{r}} L_{\mathrm{R}} = 2\pi f_{\mathrm{r}} L_{\mathrm{R}} \\ &= 2\pi(sf_e)L_{\mathrm{R}} \\ &= sX_{\mathrm{R0}} \end{aligned}$$

where X_{R0} = blocked-rotor rotor reactance.

Hence, the resulting rotor equivalent circuit is:

Rotor circuit model

The rotor current flowing is given by:

$$\bar{I}_{\mathrm{R}} = \frac{\bar{E}_{\mathrm{R}}}{R_{\mathrm{R}} + jX_{\mathrm{R}}} = \qquad\qquad =$$

From the equation above, the rotor effects due to varying rotor speed can be treated as caused by a varying impedance supplied with a power from a constant-voltage source $\bar{E}_{\mathrm{R0}}$.

Therefore, the overall equivalent rotor impedance is:

$$Z_{\mathrm{R,eq}} = \frac{R_{\mathrm{R}}}{s} + jX_{\mathrm{R0}}$$

And the rotor equivalent circuit now becomes:

Rotor circuit model with all the frequency (slip) effects concentrated in rotor impedance $\boldsymbol{Z_{\mathrm{R,eq}}}$(or resistor $\boldsymbol{R_{\mathrm{R}}}$)

In the above equivalent circuit:

- **rotor voltage** is a **constant** $\bar{E}_{\mathrm{R0}}$ V
- **rotor impedance** $Z_{\mathrm{R,eq}}$ **contains** all **effects** of **varying rotor slip**

Notice:

- at **very low slips**, $\boldsymbol{R_{\mathrm{R}}/s \gg X_{\mathrm{R0}}}$

  $\Longrightarrow$ rotor current varies linearly with slip

- at **high slips**, $\boldsymbol{X_{\mathrm{R0}} \gg R_{\mathrm{R}}/s}$

  $\Longrightarrow$ rotor current approaches steady state value

The final equivalent circuit

To obtain the **final per-phase equivalent circuit for IM**

⇨ Need to refer the rotor part of the model to the stator side

i.e. the rotor circuit with the slip effects included in R_R

Similar to the transformer, the **rotor (secondary) circuit voltages, currents and impedances** can be **referred** to the stator (primary) side **using** the **effective turns ratio, a_{eff}.**

Hence, the transformed rotor voltage is:

$$\bar{E}_1 = \bar{E}'_{\mathrm{R}} = a_{\mathrm{eff}}\bar{E}_{\mathrm{R0}}$$

the rotor current referred to the stator side:

$$\bar{I}_2 = \frac{\bar{I}_{\mathrm{R}}}{a_{\mathrm{eff}}}$$

and the rotor impedance referred to the stator side:

$$Z_2 = a_{\mathrm{eff}}^2\left(\frac{R_R}{s} + jX_{\mathrm{R0}}\right)$$

Therefore, we can define:

$$R_2 = a_{\mathrm{eff}}^2 R_{\mathrm{R}}$$

$$X_2 = a_{\mathrm{eff}}^2 X_{\mathrm{R0}}$$

Hence, the **final induction motor per-phase equivalent circuit (referred to the stator side)** can be drawn as shown below:

Note: $\boldsymbol{R}_{\mathbf{R}}$, $\boldsymbol{X}_{\mathbf{R0}}$ and $\boldsymbol{a}_{\mathbf{eff}}$ are **very difficult** to be **determined** directly **for cage rotors**. However, it is **possible** to make **measurements** to **directly give** the referred resistance $\boldsymbol{R}_{\mathbf{2}}$ **and** referred reactance $\boldsymbol{X}_{\mathbf{2}}$.

7.4. Power and torque in induction motors

Losses and power-flow diagram

The **input power (electrical)** of an induction motor:

Losses encountered on **stator side**:

- **Stator copper loss $\boldsymbol{P}_{\mathbf{SCL}}$**, i.e. I^2R loss in stator windings.
- **Hysteresis and eddy current losses $\boldsymbol{P}_{\mathbf{core}}$**

Air gap power $\boldsymbol{P}_{\mathbf{AG}}$ ⇒ power transferred to the rotor **across** the **air gap**

Losses encountered on **rotor side**:

- **Rotor copper loss $\boldsymbol{P}_{\mathbf{RCL}}$,** i.e. I^2R loss in rotor windings.

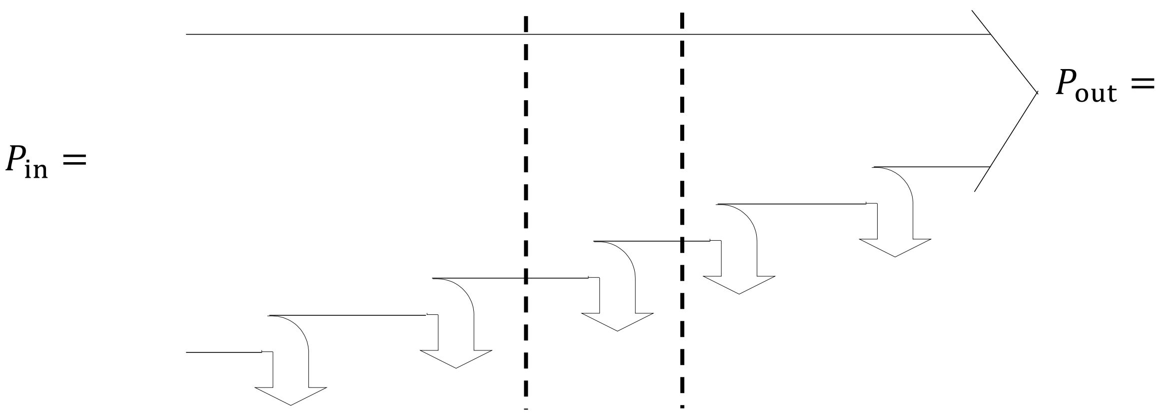
What is left?
Power converted from electrical to mechanical form, $\boldsymbol{P_{conv}}$.

Final losses:

- **Friction and windage losses, $\boldsymbol{P}_{\mathbf{F\&W}}$**
- **Stray losses, $\boldsymbol{P}_{\mathbf{misc}}$**

The **output power (mechanical)** of the **induction motor**:

Hence, the **power-flow diagram of the induction motor** is obtained:



Special note on P_{core}:

- The core losses do **not always** appear after P_{SCL}.
- P_{core} comes partially from the stator circuit and partially from the rotor circuit. Usually the rotor core losses are very small compared to the stator core losses.
- P_{core} are represented in the induction motor equivalent circuit by the **resistor R_C** (or the conductance G_C).
- If R_C is **not given** but **P_{core} = X watts** is **given**, then often **add it together with $P_{F\&W}$** at the end of the power flow diagram.

  Note: **P_{core}, $P_{F\&W}$**, and **P_{misc}** are sometimes lumped together and called **rotational losses P_{rot}**.

<u>Example</u>

A 480V, 60Hz, 37.3-kW, 3 phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2kW, and the rotor copper losses are 700W. The friction and windage losses are 600W, the core losses are 1800W, and the stray losses are negligible. Find:

a) The air gap power P_{AG}
b) The power converted P_{conv}
c) The output power P_{out}
d) The efficiency of the motor

Power and torque in an induction motor

The **power and torque equations** governing the motor operation **can** be **derived from** the per phase **equivalent circuit** of an induction motor.

Input current to a motor phase is:

$$I_1 = \frac{V_\Phi}{Z_{\mathrm{eq}}}$$

where :

$$Z_{\mathrm{eq}} =$$

The **three-phase stator copper losses**:

$$\boxed{P_{\mathrm{SCL}} = \qquad\qquad\qquad\qquad}$$

The **three-phase core losses**:

$$\boxed{P_{\mathrm{core}} = \qquad\qquad\qquad\qquad}$$

Therefore, the **air gap power** can be found using:

$$\boxed{P_{\mathrm{AG}} = 3I_2^2 \frac{R_2}{s}}$$

Note: This is because the air gap power can **only** be **consumed** by the **resistor** $\boldsymbol{\frac{R_2}{s}}$

The tactual hree-phase resistive **rotor copper losses**:

$$\boxed{P_{\mathrm{RCL}} = 3I_2^2 R_R = 3I_2^2 R_2}$$

Note: The last term in the rotor copper loss equation is due to the fact that **power** is **unchanged when referred across** an **ideal transformer**.

The **power converted from electrical to mechanical form (**or ***developed mechanical power*****)** is given by:

$$P_{\text{conv}} = P_{\text{in}} - P_{\text{SCL}} - P_{\text{core}} - P_{\text{RCL}}$$
$$P_{\text{conv}} = P_{\text{AG}} - P_{\text{RCL}} = 3I_2^2\frac{R_2}{s} - 3I_2^2R_2$$

Notice that:

$$\boldsymbol{P_{\text{RCL}} = sP_{\text{AG}}}$$

Hence,

- as the **slip reduces** $\Rightarrow$ the **rotor losses reduce**
- When **rotor stationary, *s* = 1**:
  $\Rightarrow$ air gap power is **entirely consumed** by rotor

Therefore, another expression for power converted is:

$$\boldsymbol{P_{\text{conv}} = (1-s)P_{\text{AG}}}$$

If the friction and windage losses and stray losses are known, the **output power** is:

$$\boldsymbol{P_{\text{out}} = P_{\text{conv}} - P_{\text{F\&W}} - P_{\text{misc}}}$$

As for the **induced torque $\boldsymbol{\tau_{\text{ind}}}$** (or ***developed torque***) in the machine is:

Note: $\boldsymbol{\tau_{\text{ind}}}$ is the torque generated by the electrical-to-mechanical power conversion. An alternative expression for $\boldsymbol{\tau_{\text{ind}}}$:

$$\boldsymbol{\tau_{\text{ind}} = \frac{(1-s)P_{\text{AG}}}{(1-s)\omega_{\text{sync}}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}}$$

Separating the rotor copper losses and power converted in an induction motor's equivalent circuit

Power crossing the air gap, P_{AG}

Rotor copper losses, P_{RCL}

Converted mechanical power, P_{conv}

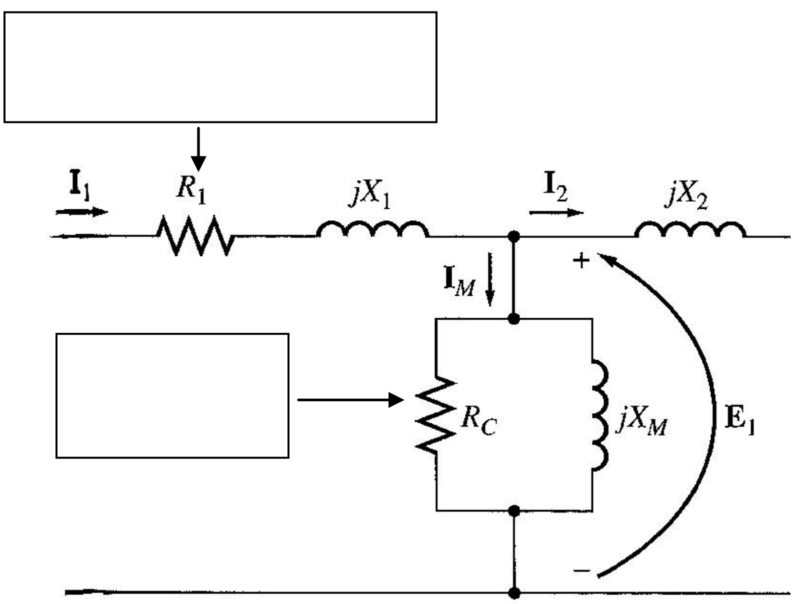
It is possible to indicate this separately on the motor equivalent circuit.

$\boldsymbol{P}_{\mathbf{AG}}$ is **consumed b**y the resistor: $\boldsymbol{\frac{R_2}{s}}$
$\boldsymbol{P}_{\mathbf{RCL}}$ is **consumed by** the resistor: $\boldsymbol{R_2}$

Therefore, $\boldsymbol{P}_{\mathbf{conv}}$ **=** $\boldsymbol{P}_{\mathbf{AG}}$ **-** $\boldsymbol{P}_{\mathbf{RCL}}$ must be **consumed in** a **resistor** of value:

$$\boldsymbol{R}_{\mathbf{conv}} = \frac{\boldsymbol{R_2}}{\boldsymbol{s}} - \boldsymbol{R_2} = \boldsymbol{R_2}\left(\frac{\mathbf{1}}{\boldsymbol{s}} - \mathbf{1}\right)$$

Hence, the **induction motor per-phase equivalent circuit** can be **modified** to be:



Example

A 460V, 25hp, 60Hz, 4 pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$ $R_2 = 0.332\ \Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_m = 26.3\ \Omega$

The total rotational losses are 1100W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2% at the rated voltage and rated frequency, find the motor's

a) speed
b) stator current
c) power factor
d) P_{conv} and P_{out}
e) τ_{ind} and τ_{load}
f) efficiency